Teacher Guide

for

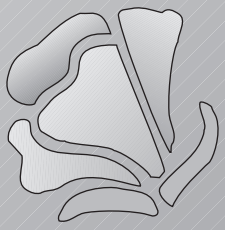
First-year Algebra

and

Geometry

•

2003



Golden State Examination

GSE

CALIFORNIA DEPARTMENT OF EDUCATION

This document has been prepared by the Sacramento County Office of Education and San Joaquin County Office of Education, under contract with the California Department of Education. For information about the Golden State Examination testing dates, registration materials and procedures, or the Golden State Seal Merit Diploma, contact:

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Table of Contents

Introduction

Using the Golden State Examination Teacher Guide

The *Golden State Examination Teacher Guide* has been developed to provide essential information and preparation guidelines for teachers and to serve as an instructional aid in the classroom. It is divided into the following sections:

Test Content — outlines state content standards for which Golden State Examination (GSE) questions are developed.

Test Structure — describes the format of the test.

Scoring Guide — outlines the standards-based criteria used to score written-response problems.

Sample Questions — includes sample questions that represent types of questions found on the exams.

Student Work — provides examples of student responses to written-response problems with teacher commentary.

Teachers are encouraged to reproduce portions or all of the guide for classroom use. Districts/schools also may use these materials with state standards for staff development.

Student Eligibility

The first-year algebra and geometry exams are given in the spring. Each examination may be taken only once.

Students who are enrolled in first-year algebra (Algebra I) or geometry at the time the examinations are given and students who have taken the courses since the last test administration may take the exams. Students completing the second year of a two-year Algebra I course may take the first-year algebra exam. Students completing a second-year integrated mathematics course may take either the first-year algebra exam, the geometry exam, or both.

Test Preparation

The Golden State Examinations in mathematics are aligned to state content standards. Teachers should review their curriculum and instructional activities for alignment to these standards.

Sound preparation for the Golden State Examinations should include classroom assignments that allow students to articulate the major ideas and concepts in the subject area being tested. Students also must be able to analyze information, apply knowledge, solve problems, and explain their solutions.

Note: It also is important that students and parents receive information about the testing dates and ways to prepare for the exams well in advance.

Special Accommodations

Accommodations for testing as specified in a student's Individualized Education Program (IEP) or 504 plan apply to GSE administration. For students with visual impairments, a copy of the examination to be administered can be sent to the district for enlargement or reproduction in braille. Please allow sufficient time for this process. Contact NCS Pearson, the GSE testing contractor, at (800) 627-7990 ext. 805 for more information, questions, and/or arrangements for special testing needs or situations.

Reporting Results

Currently, the Golden State Examinations in First-year Algebra and Geometry each consist of two 45-minute sessions. Students who complete both sessions receive an individual report of results. Scores for the multiple-choice and written-response portions of the exam are combined to produce the student's overall achievement level. There are six achievement levels: high honors (6), honors (5), recognition (4), and acknowledgment for participation (3, 2, and 1). Results of the spring administration are mailed to districts in October.

Resource Documents

The *Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve*, and the *Mathematics Frameworks for California Public Schools, Kindergarten Through Grade Twelve* are available at the California Department of Education, CDE Press, Sales Office, P.O. Box 271, Sacramento, CA 95812-0271; 1-800-995-4099 ext. 6. These documents also are available at http://www.cde.ca.gov/board on the Internet.

Other Resources

Testing schedules and other information are available from your district GSE coordinator, your county office of education, or the California Department of Education at http://www.cde.ca.gov/statetests/gse on the Internet.

Test Content for First-year Algebra and Geometry

The content of the Golden State Examinations in First-year Algebra and Geometry is aligned to the *Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve*. A complete listing of the mathematics standards and the *Mathematics Frameworks for California Public Schools, Kindergarten Through Grade Twelve* are available at http://www.cde.ca.gov/board on the Internet.

First-year Algebra

The GSE in First-year Algebra measures the use and application of basic algebraic skills and students' justifications in solving problems. Algebra I standards are the focus of the content. Other standards are addressed when they serve as a foundation for the algebraic concept assessed. Topics covered in the examination include but are not limited to the following content:

Content	Algebra I Standards
Identify and use the arithmetic properties of integers, rational, irrational, and real numbers for evaluation and simplification of algebraic expressions, equations, and inequalities using:	
• taking a root, exponents, opposites, reciprocals	2.0
• factoring	12.0, 13.0
• monomials, polynomials	10.0, 12.0
Solve equations, inequalities, and systems of equations using:	
• linear and quadratic equations	3.0, 4.0, 16.0, 5.0, 10.0, 13.0, 15.0, 9.0
• factoring up to third-degree polynomials	11.0, 22.0
• completing the square, quadratic formula	11.0, 14.0, 19.0, 20.0, 23.0
• absolute value, proportion	3.0, 5.0, 10.0, 13.0, 15.0
Graph linear and quadratic equations, inequalities, and functions involving:	
• parallel and perpendicular relationships, intercepts, intersection	6.0, 8.0, 21.0, 22.0, 9.0
• distance and midpoint formulas, domain and range	7.0, 17.0
• alternative forms of linear equations, inequalities, making predictions	25.0
Apply the knowledge and skills of algebra, using appropriate problem-solving strategies.	1.0, 10.0, 13.0, 15.0, 23.0, 24.0, 25.0

Geometry

Computation, use of algebra, problem solving, proofs (formal and informal), and applications are integrated throughout the GSE in Geometry. Geometry standards are the focus of the content. Other standards are addressed when they serve as a foundation for the geometric concept assessed. Topics covered in the examination include but are not limited to the following content:

Content	Geometry Standards	Trigonometry Standards
Identify relationships among angles, lines, planes, and exterior angles related to:		
• triangle inequalities	6.0	
• parallel lines, intersecting lines, polygons	7.0	
• angles, lines, planes, exterior angles	12.0, 1.0	
Identify and apply triangle and trigonometric relationships:		
• Pythagorean relationships, similarity, congruence	14.0, 15.0, 5.0, 6.0, 4.0	
• special triangle relationships	20.0, 12.0, 13.0	
• area and perimeter of triangles	8.0, 10.0	
• ratios: sine, cosine, tangent, law of sines	18.0, 19.0	12.0, 13.0, 19.0

Content	Geometry Standards
Use properties of polygons other than triangles, involving:	
• quadrilaterals	7.0, 10.0, 12.0
• polygons with five or more sides	10.0, 12.0
• relationships within polygons; sides, angles, midpoints, diagonals	13.0, 17.0
• area, perimeter, similarity	11.0, 4.0
Identify and apply properties of circles, involving:	
• angle and segment relationships	7.0, 12.0, 21.0
• basic relationships involving circles, area, circumference, sectors, arc measure, arc length	8.0, 11.0, 17.0
Relate properties of coordinate and transformational geometry, including:	
• linear, nonlinear	15.0
• translations, reflections, rotations, dilations	22.0
Solve problems using the properties of three dimensional figures (including angles, surface area and volume of prisms, pyramids, cylinders, cones, and spheres)	8.0, 9.0, 11.0, 1.0, 2.0, 3.0

Test Structure for First-year Algebra and Geometry

The Golden State Examinations in First-year Algebra and Geometry are currently two-part examinations, administered in 45-minute sessions. Each of the two sessions consists of multiple-choice questions and a written-response problem.

The multiple-choice questions are designed to assess the student's breadth of knowledge. The questions emphasize concepts, principles, analysis, and the application of basic processes. The multiple-choice questions may require students to make connections among mathematical concepts or to organize information to arrive at the correct answer.

The multiple-choice portion of the examinations is machine-scored. Sample multiple-choice questions and answer keys are on pages 9–11 for first-year algebra and pages 19–22 for geometry.

The written-response problems require students to apply their mathematical skills and knowledge. Students are given problems and asked to provide a correct solution with all steps of their solution clearly shown. Students also are required to provide an explanation of how they arrived at the solution.

The written-response portion of the examinations is scored by experienced mathematics teachers and other professionals in the field. Sample written-response problems with student work and teacher commentary are presented on pages 12–18 for first-year algebra and pages 23–31 for geometry.

Teachers are encouraged to duplicate this guide for student use and to have students test themselves with the sample questions and problems. State content standards addressed by the written-response problems are identified for the purpose of this guide but do not appear on the examination.

Calculators may no longer be used for the Golden State Examinations in mathematics.

Golden State Examinations—California Standards Tests

Education Code section 60650 now requires Golden State Examinations (GSE) to be administered as an augmentation to the California Standards Tests (CST) unless there is no CST in the subject area being tested. *Education Code section 60653* requires the GSE to consist of some portion of the CST and additional GSE items in order to reduce testing time in subjects for which a GSE and a CST exist.

It is anticipated that the GSE in Geometry will be administered as an augmentation to the CST in 2004. Information about the format of the exam will be included in the 2004 teacher guide.

It is anticipated that the GSE in First-year Algebra will be administered as an augmentation to the CST in 2005.

Scoring Guide for First-year Algebra and Geometry

The written-response portion of the Golden State Examinations in First-year Algebra and Geometry is scored in three components: Conceptual Understanding, Mathematical Computation and Accuracy, and Communication. The general scoring guides for the three components are given below.

Conceptual Understanding Component 1

Score Point 4

The student response demonstrates thorough understanding of the central mathematical concepts. The response:

- shows a thorough understanding of the central mathematical ideas as demonstrated by a correctly identified and applied process.

Score Point 3

The student response demonstrates substantial understanding of the central mathematical concepts. The response:

- shows substantial understanding of the central mathematical ideas as identified by an appropriate process that contains a flawed application.

Score Point 2

The student response demonstrates partial understanding of the central mathematical concepts. The response:

- shows limited understanding of the central mathematical ideas by showing a process that could lead to a correct answer.

Score Point 1

The student response demonstrates little or no understanding of the central mathematical concepts. The response:

- shows little or no understanding or uses an invalid process.

Mathematical Computation and Accuracy Component 2

Score Point 4

The student response accurately demonstrates all required mathematical computations. The response:

- shows thorough use of mathematical skills by accurately demonstrating all required computations.

Score Point 3

The student response demonstrates general accuracy in mathematical computations. The response:

- shows substantial use of mathematical skills; may include minor computational errors, such as transposition of digits, or incorrect copying.

Score Point 2

The student response demonstrates partial accuracy in required mathematical computations. The response:

- shows limited use of mathematical skills; may include numerous computational errors.

Score Point 1

The student response demonstrates little or no accuracy in required mathematical computations. The response:

- shows little or no use of mathematical skills required for this prompt.

Communication Component 3

Score Point 3

The student response demonstrates clear communication of mathematical ideas. The response:

- shows clear organization with supporting evidence.
- uses appropriate mathematical language and reasoning.

Score Point 2

The student response demonstrates partial communication of mathematical ideas. The response:

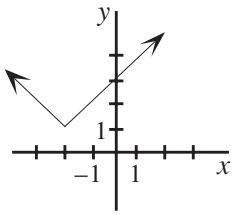
- shows some organization with limited supporting evidence.
- may misuse or contain inappropriate mathematical language.

Score Point 1

The student response demonstrates little or no communication of mathematical ideas. The response:

- shows little or no organization or provides little or no supporting evidence.
- contains little or no mathematical language; may not be comprehensible.

Sample Multiple-choice Questions for First-year Algebra

1. 

Which of the following equations represents the graph above?

A. $y = -|x + 2| + 1$

B. $y = |x + 2| + 1$

C. $y = |x - 2| - 1$

D. $y = |x + 2| - 1$

2. What is the y-intercept of a line that passes through (–5, 6) and has an x-intercept of 3?

A. $\frac{4}{9}$ B. 2

C. $2\frac{1}{4}$ D. 3

3. Of the following three polynomials, which are not factorable?

I. $x^2 + 2x + 18$

II. $x^2 + 4x$

III. $x^2 + 10x + -96$

A. I only B. III only

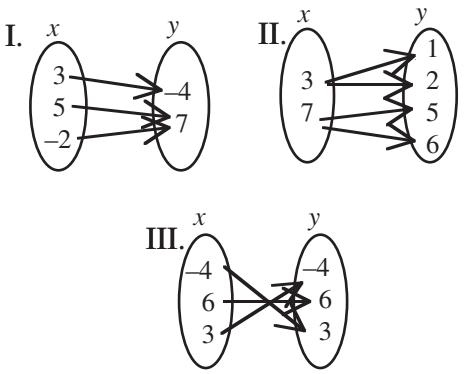
C. I and III only D. II and III only

4. Solve $|3x + 7| = 11$.

A. -6 only B. $\frac{4}{3}$ only

C. $\frac{4}{3}$ and $-\frac{4}{3}$ D. $\frac{4}{3}$ and -6

5. Using the following mapping, which represent functions?



A. I only

B. II only

C. I and III only

D. II and III only

6. Graciela wanted to buy a new winter coat. The store was offering a 30% discount plus an extra $6 discount for paying cash. Graciela bought a coat for cash and paid $80.80. Which of the following equations could be used to find the **original price** of the coat, x?

A. $0.70x + 6 = 80.80$

B. $0.70x - 6 = 80.80$

C. $x - 0.30(x + 6) = 80.80$

D. $x - 0.30(x - 6) = 80.80$

7.

x	-1	2	3	7
y	33	21	17	1

From the chart above, what is the value of y when $x = 12$?

A. 68 B. 19

C. –19 D. –48

8. How many different values of k exist so $x^2 + kx - 15$ is factorable using integers?

A. 1 B. 2

C. 4 D. 6

9. Pipe A can fill a tank in 8 hours. When both pipe A and pipe B are used, the tank is filled in 5 hours. How long, in hours, would it take pipe B alone to fill the tank?

A. $6\frac{1}{2}$ B. 9

C. 13 D. $13\frac{1}{3}$

10. Jack and Jill sat at the ends of an 18-foot long seesaw where the fulcrum was placed in the middle of the board. By placing a 20-pound weight 3 feet in front of Jill, they were in balance. If Jack weighs 140 pounds, which equation could be used to determine Jill's weight (x) in pounds?

A. $140(9) = 9x + 6(20)$

B. $140(9) = 9x + 3(20)$

C. $140(9) = 9x + (x - 3)20$

D. $140(18) = 18x + 15(20)$

11. When $\frac{p+2}{4p} - \frac{3p-1}{6p^2}$ is simplified, the numerator is:

A. $3p^2 - 2$ B. $4p + 1$

C. $3p^2 + 2$ D. $-2p + 3$

12. Solve for t:

$$2(t + 3) - 3(-2t + 5) = \frac{1}{2}(16t - 18)$$

A. {0}

B. {9}

C. {real numbers}

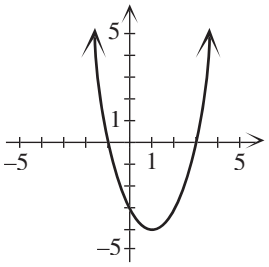
D. no real solution

13. What is the value of c if $x^2 + 7x + c$ is a perfect square?

A. $\sqrt{7} \approx 2.65$ B. $\frac{7}{2}$

C. $\frac{49}{4}$ D. 49

14.



The figure shows the graph of $y = x^2 - 2x - 3$. The expression $x^2 - 2x - 3$ is less than zero when:

A. $x < 0$

B. $x < 1$ or $x > 3$

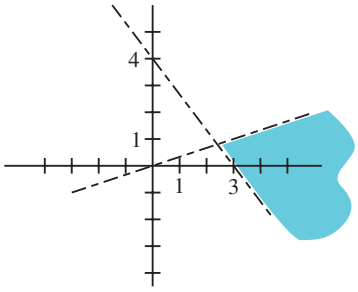
C. $x > -4$

D. $-1 < x < 3$

15. The vertex of the parabola represented by $y = x^2 + 8x + 13$ is in which quadrant?

A. I
B. II
C. III
D. IV

16.



The shaded region is determined by which pair of inequalities?

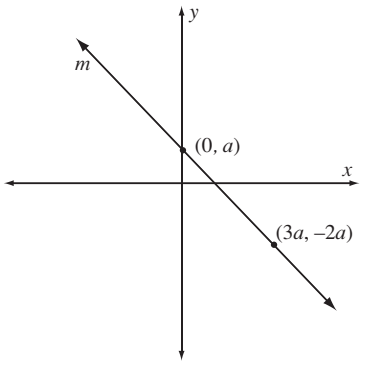
A. $4x + 3y < 12$
$x - 3y < 0$

B. $4x + 3y < 12$
$x - 3y > 0$

C. $4x + 3y > 12$
$x - 3y < 0$

D. $4x + 3y > 12$
$x - 3y > 0$

17.



Note: Figure not drawn to scale.

Which of the following is an equation for line m?

A. $y = -ax + a$

B. $y = -x + a$

C. $y = \frac{-3}{2}x + a$

D. $y = \frac{-3}{2}ax + a$

First-year Algebra Answer Key

1. B	4. D	7. C	10. A	13. C	16. D
2. C	5. C	8. C	11. C	14. D	17. B
3. A	6. B	9. D	12. C	15. C	

Sample Written-response Problem for First-year Algebra

Your work for the following question will be evaluated on accuracy as well as on clarity of the presentation of the solution.

A complete response includes:
1. your answer to the question
2. your work CLEARLY shown
3. use of an equation (or equations) when appropriate
4. use of a diagram or graph when appropriate
5. all parts of your solution shown
6. an explanation of how you arrived at your answer

Tom wrote $5x - 27 = 9 - 4x$ and said he was going to solve it. Alison looked at the equation and wrote $\begin{cases} y = 5x - 27 \\ y = 9 - 4x \end{cases}$. She claimed she could graph this pair of equations and get the same solution as Tom.

a. Show how Tom solved his equation.

b. Graph Alison's equations and explain her solution.

c. Who is correct? Justify your conclusion.

Algebra I Standards – 4.0, 5.0, 6.0, 9.0

What Students Are Expected to Accomplish Mathematically

To be correct, student responses should include a correct solution to Tom's equation for $x = 4$, Alison's graph of the system of equations with the intersection of the two lines at (4, –7), and a justification for both answers. Students must clearly and accurately apply the processes.

Sample Student Work for First-year Algebra

Conceptual Understanding: 4
Mathematical Computation and Accuracy: 4
Communication: 3

STUDENT RESPONSE*

TOM: 5x – 27 = 9 – 4x
(A) 9x = 36
x = 4

you need to get the x alone. This will tell you what x equals.

x = 4

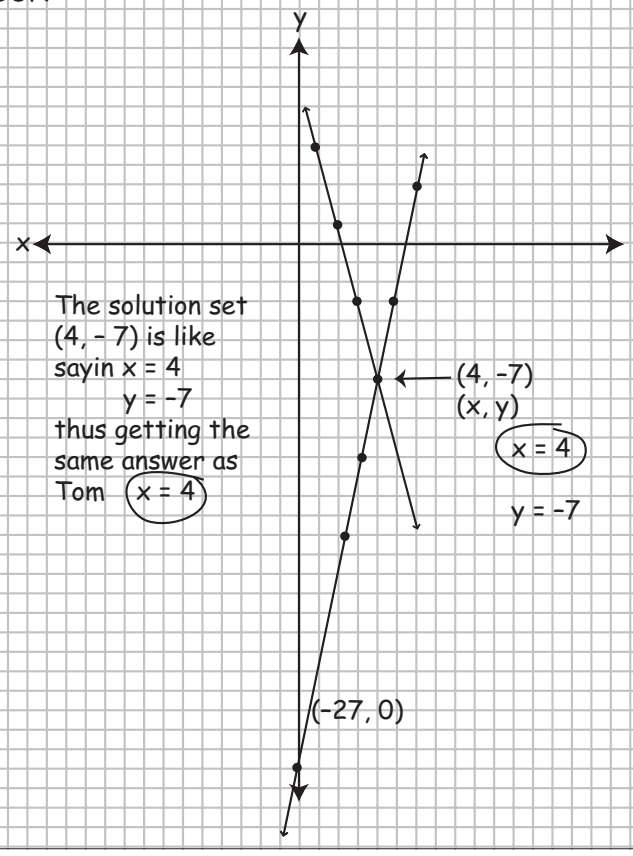
5(4) – 27 = 9 – 4(4)
20 – 27 = 9 – 16 – 7 = – 7

EXPLANATION:

(C)

Both ways are correct. Both Allison and Tom got the answer (4) for "X." Tom's way is more direct because he gets the answer immediately, while Allison gets a solution set. But, she still gets (4) as "X." The solution set (4, – 7) is the same as (x = 4.) It is the intersection. y = – 7.

ALLISON
(B)



* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

COMMENTARY

The response demonstrates a thorough understanding of the central mathematical concepts by providing a solution to Tom's equation, a thorough graph of Alison's system, and an analysis of their sameness.

The response has the correct solution $x = 4$, correct point of intersection (4, –7), and correct analysis that they have the same x value.

The methods of solving are complete and easy to follow.

Sample Student Work for First-year Algebra

Conceptual Understanding: 4
Mathematical Computation and Accuracy: 3
Communication: 3

STUDENT RESPONSE*

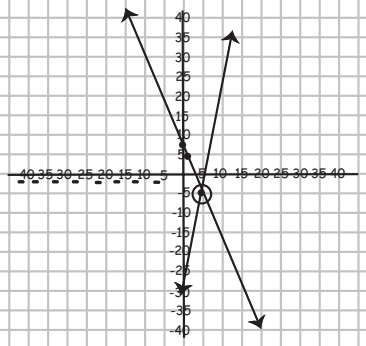
a) $5x - 27 = 9 - 4x$

$+4x \quad +27 \quad +27 \quad +4x$

$9x = 36$

$x = 4$

b)



$y = 5x - 27$

x	y
0	-27
1	22
2	17
3	12
5	

$y = 9 - 4x$

x	y
0	2
1	5
2	1
3	-3

c. Both Alisn and Tom are correct. The answer, x = 4, can be found using both methods.

By solving the equation you can find that x = 4.

When you graph Allison's equation, the two lines intersect at x = 4.

COMMENTARY

The response demonstrates a thorough understanding of the central mathematical concepts by providing a solution to Tom's equation, a thorough graph of Alison's system, and an analysis of the relationship between the two methods.

The response includes Tom's equation solved correctly and Alison's system graphed correctly, but the scale on the graph cannot verify an accurate solution of (4, – 7).

The response clearly communicates all required parts of the prompt.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

Conceptual Understanding: 4
Mathematical Computation and Accuracy: 3
Communication: 3

STUDENT RESPONSE*

A) 5x – 27 = 9 – 4x (Tom's equation solved.)

+ 4x + 4x

9x – 27 = 9

+27 +27

9x = 36

÷ 9 ÷ 9

x = 4

5(4) – 27 = 9

y = – 7

9 – 4(4) =

9 + - 16 = – 7

y = – 7

* With tom's method they would intersect at (4, – 7)

Line (1) 5x – 27 = y Line (2) 9 – 4x = y

B) y = 5x – 27

y = 9 – 4x

Alison's solution was the same as Tom's was. (4, – 7) intercept

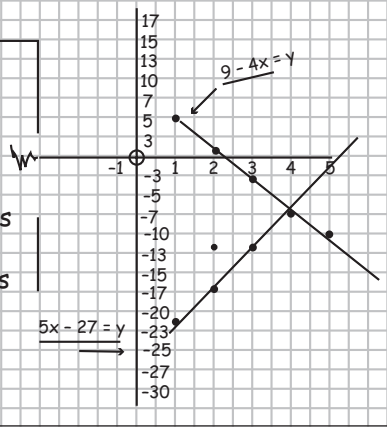
x	y
1	-22
2	-17
3	-12
4	-7
5	-2

Table

x	y
1	5
2	1
3	-3
4	-7
5	-1

Table

C) They are both correct. In both cases I got the same answers. The only difference was the method with which they chose to solve the equations The answers were identical but solving the equation was faster.



COMMENTARY

The response demonstrates a thorough understanding of the mathematical concepts central to the prompt. The response includes the linear equation solved and the system of equations graphed.

Tom's equation is solved correctly and a correct table is written for both lines graphed. The point of intersection is shown on the graph, but the scale used on the graph is inaccurate.

The response is clearly presented with supporting evidence, including appropriate mathematical language and reasoning as well as a justification for both methods.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

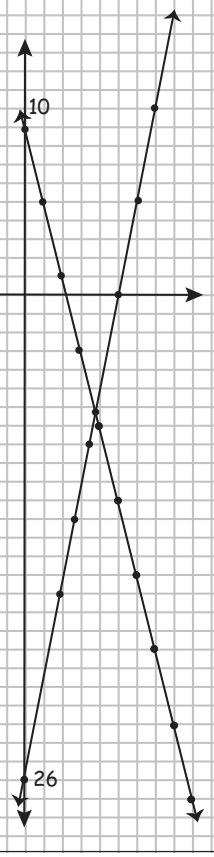
Conceptual Understanding: 3
Mathematical Computation and Accuracy: 2
Communication: 3

STUDENT RESPONSE*

a.) 5x – 27 = 9 – 4x
9x = 36
x = 4
You move the common #s to one side of the equation and divide

b.) y = 5x – 27
y = – 4x + 9

c.) Allison is correct because the two lines intersect at 4, which is the answer to tom's equation



COMMENTARY

The response demonstrates a substantial understanding of the mathematical concepts by solving Tom's equation for x and graphing Alison's solution.

The response includes a correct solution to Tom's equation and uses the slope to plot Alison's system. There is inconsistency in the graphed solution with the statement made. The lines do not intersect at $x = 4$ due to a graphing error.

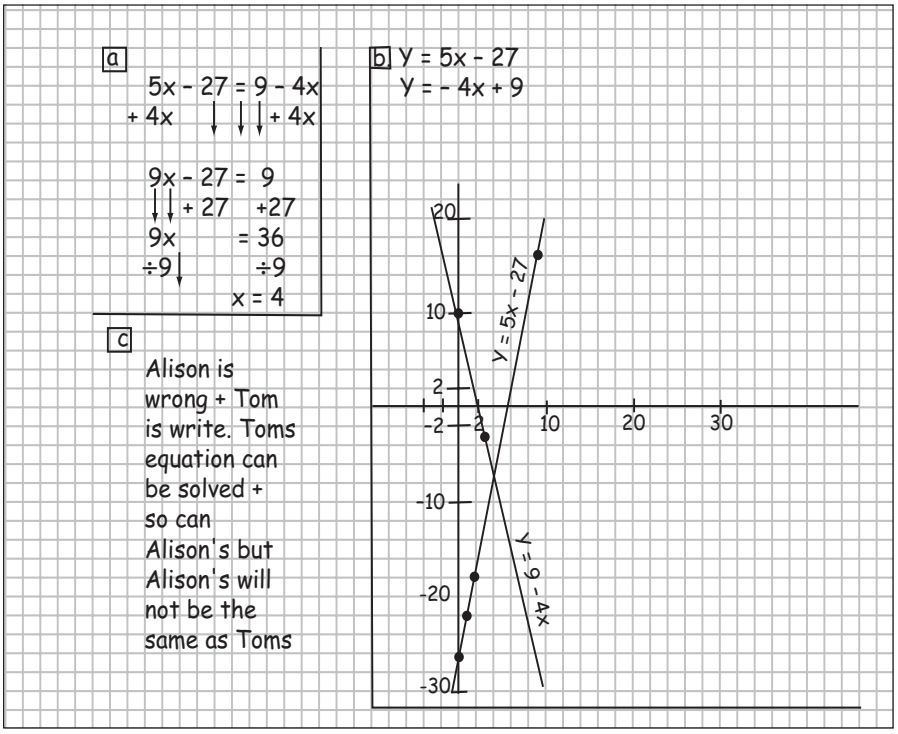
The response shows supporting evidence with solutions to both and makes the connection between the two methods used.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

Conceptual Understanding: 2
Mathematical Computation and Accuracy: 2
Communication: 2

STUDENT RESPONSE*



COMMENTARY

The response solves the linear equation and graphs the system of equations without identifying the point of intersection and has an invalid conclusion.

The linear equation is solved correctly, but there is no point of intersection labeled on the graph.

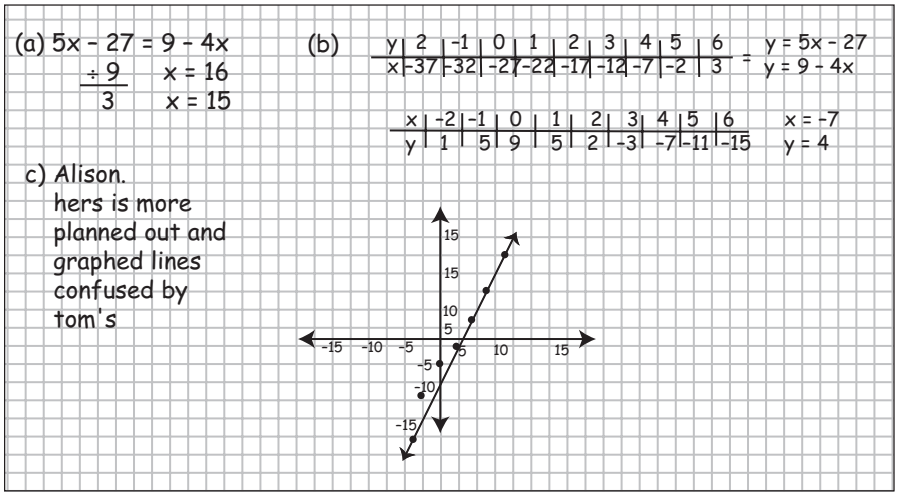
The student response demonstrates partial communication of mathematical ideas. The point of intersection is not labeled and there is limited evidence for the conclusion.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for First-year Algebra

Conceptual Understanding: 1
Mathematical Computation and Accuracy: 1
Communication: 2

STUDENT RESPONSE*



COMMENTARY

The response demonstrates little understanding of the mathematical concepts required. The response includes the point of intersection from an appropriate table without solving Tom's equation.

The response does not solve Tom's equation and does not graph the system of equations.

The response addresses all required parts of the prompt.

Sample Multiple-choice Questions for Geometry

1. Two planes intersect at line $\overleftrightarrow{AB}$. Each plane contains a line that does not intersect $\overleftrightarrow{AB}$. Which of the following terms describes the relationship between these two lines?

   A. skew

   B. coplanar

   C. perpendicular

   D. intersecting

2. ΔMNO is similar to ΔXYZ. If $MN = 15$, $MO = 20$, $NO = 30$, $XY = 4k - 2$, and $XZ = 4k + 4$, find XY.

   A. 6 B. 12

   C. 15 D. 18

3. Line AE intersects line CD at point B. Point B lies between A and E and between C and D. $m\angle ABC = (x - 2)(2x + 2)$, and $m\angle DBE = (x - 1)(x + 4)$. What is the measure of $\angle CBE$?

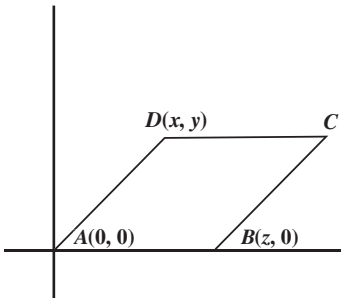
   A. 5 B. 36

   C. 144 D. 176

4. In parallelogram $ABCD$, the measure of $\angle A$ is $(2x + 5)°$ and the measure of $\angle C$ is $(4x - 59)°$. The measure of $\angle B$ is:

   A. 32° B. 69°

   C. 83° D. 111°

5. 

   Note: Figure not drawn to scale.

   Figure $ABCD$ is a rhombus, with vertices located at the coordinates shown. What is the slope of a line perpendicular to side $\overline{BC}$?

   A. $\frac{x}{y}$ B. $\frac{y}{x}$

   C. $-\frac{x}{y}$ D. $-\frac{y}{x}$

6. Which of the following points are vertices of a square with one side tangent to the circle $(x - 3)^2 + y^2 = 9$?

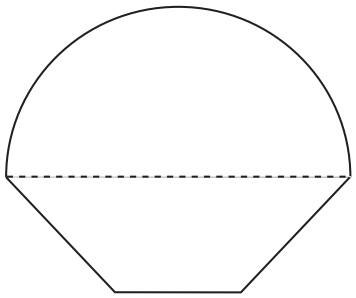
   A. (0, 0) (6, 0)
   (3, 3) (3, –3)

   B. (– 4, –1) (0, 1)
   (– 4, 1) (0, –1)

   C. (6, 2) (10, 2)
   (6, –2) (10, –2)

   D. (6, 6) (9, 3)
   (6, 0) (3, 3)

7.



Note: Figure not drawn to scale.

The floor of a theater is made up of a semicircle with radius 50 feet and one-half of a regular hexagon as shown. How much carpet should be purchased to cover this floor? All answers are in square feet.

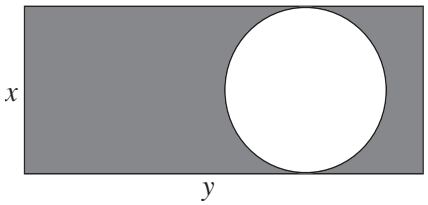
A. $1250\pi + 625\sqrt{3}$

B. $1250\pi + 1875\sqrt{3}$

C. $2500\pi + 1875\sqrt{3}$

D. $2500\pi + 3750\sqrt{3}$

8.



Note: Figure not drawn to scale.

A round ice skating rink is built inside a rectangular arena as shown in the figure. The management needs to order new material to cover the floor represented by the shaded area. Which of the following algebraic expressions represents this shaded area?

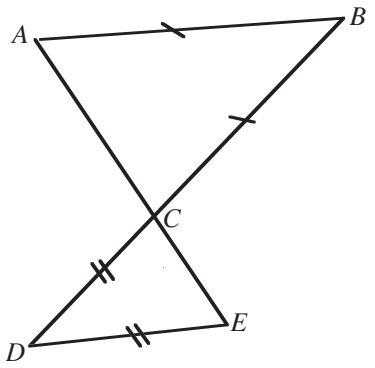
A. $xy - \pi x^2$

B. $\pi x^2 - xy$

C. $xy - \dfrac{\pi x^2}{4}$

D. $xy - \dfrac{\pi x^2}{2}$

9.



Note: Figure not drawn to scale.

What is the measure of $\angle B$ if $m\angle A = 4x$, $m\angle D = 6x - 2$, $\overline{AB} \cong \overline{BC}$, and $\overline{DC} \cong \overline{DE}$?

A. 52

B. 67

C. 72.8

D. 76

10. The endpoints of one diagonal of a square are (–5, 2) and (7, – 6). What is the equation of the line containing the other diagonal?

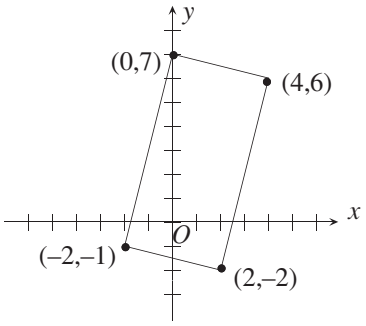
A. $3x + 2y = -1$

B. $3x + 2y = 2$

C. $3x - 2y = -1$

D. $3x - 2y = 7$

11. The area of the rectangle shown below is:



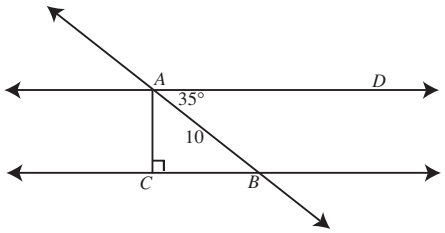
A. $6\sqrt{5} \approx 13.42$

B. $2\sqrt{85} \approx 18.44$

C. 32

D. 34

12.



Note: Figure not drawn to scale.

$\overleftrightarrow{AD} \parallel \overleftrightarrow{CB}$ and cut by transversal $\overline{AB}$. $\overline{AC} \perp \overleftrightarrow{CB}$. AB = 10, $\angle DAB = 35°$. Which of the following expressions could be used to determine the length of $\overline{AC}$?

A. 10 cos 55°

B. 10 sin 55°

C. 10 cos 35°

D. 10 tan 35°

13. What is the area, in square inches, of the circle that is circumscribed about an equilateral triangle that has a perimeter of 36 inches?

A. $18\pi \approx 56.55$

B. $27\pi \approx 84.82$

C. $36\pi \approx 113.10$

D. $48\pi \approx 150.80$

14. The apothem (drawn from the center, perpendicular to a side) of a regular hexagon has a length of 6. What is the perimeter of the hexagon?

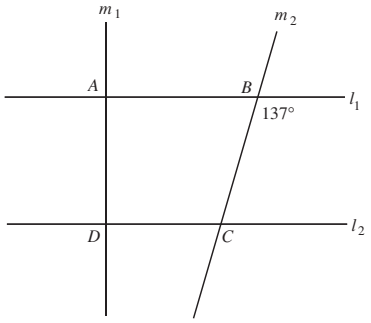
A. $2\sqrt{3} \approx 3.46$

B. $4\sqrt{3} \approx 6.93$

C. $12\sqrt{3} \approx 20.78$

D. $24\sqrt{3} \approx 41.57$

15.



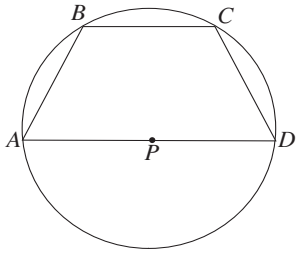
Note: Figure not drawn to scale.

In the figure $l_1 \parallel l_2$, m_1 and m_2 are transversals, and $m_1 \perp l_2$. If AD = 15.8 cm, what is BC in cm?

A. 10.8 B. 16.9

C. 21.6 D. 23.2

16.



Note: Figure not drawn to scale.

Trapezoid $ABCD$ is inscribed in circle P. If $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ and AP = 12, what is the measure of the median of the trapezoid?

A. 6 B. 12

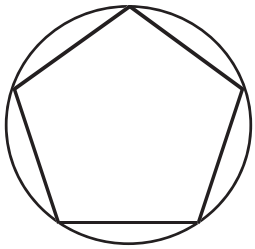
C. 18 D. 24

17. First rotate the triangle bounded by the vertices (0, 0), (0, 5), (4, 0) about the origin 180°. Then, reflect the resulting image across the x-axis. What quadrant contains the final image of the triangle after both transformations?

A. quadrant I B. quadrant II

C. quadrant III D. quadrant IV

18.



Note: Figure not drawn to scale.

A circle of area 25π circumscribes a regular pentagon as shown. To the nearest tenth, what is the length of an arc joining two adjacent vertices?

A. 4.3 B. 5.0

C. 5.9 D. 6.3

Geometry Answer Key

1. B	4. D	7. B	10. D	13. D	16. C
2. D	5. C	8. C	11. D	14. D	17. B
3. C	6. C	9. D	12. A	15. D	18. D

Sample Written-response Problem for Geometry

Your work for the following question will be evaluated on accuracy as well as on clarity of the presentation of the solution.

A complete response includes:
1. your answer to the question
2. your work CLEARLY shown
3. use of an equation (or equations) when appropriate
4. use of a diagram or graph when appropriate
5. all parts of your solution shown
6. an explanation of how you arrived at your answer

A preschool is planning to fence in a playground in the shape of a hexagon *ABCDEF*. When a scaled version of the playground is placed on a coordinate axis, the vertices are $A(1, 9)$, $B(5, 9)$, $C(5, 3)$, $D(9, 1)$, $E(9, -5)$, and $F(1, 1)$. After completing the scaled version, the preschool realizes that by connecting vertices *B* and *D* with some additional fencing they can enclose a separate triangular region for storage. If 1 unit on the coordinate axis represents 5 feet, how many feet of fencing is required for the playground and additional region?

Geometry Standard – 8.0, 11.0, 12.0, 15.0

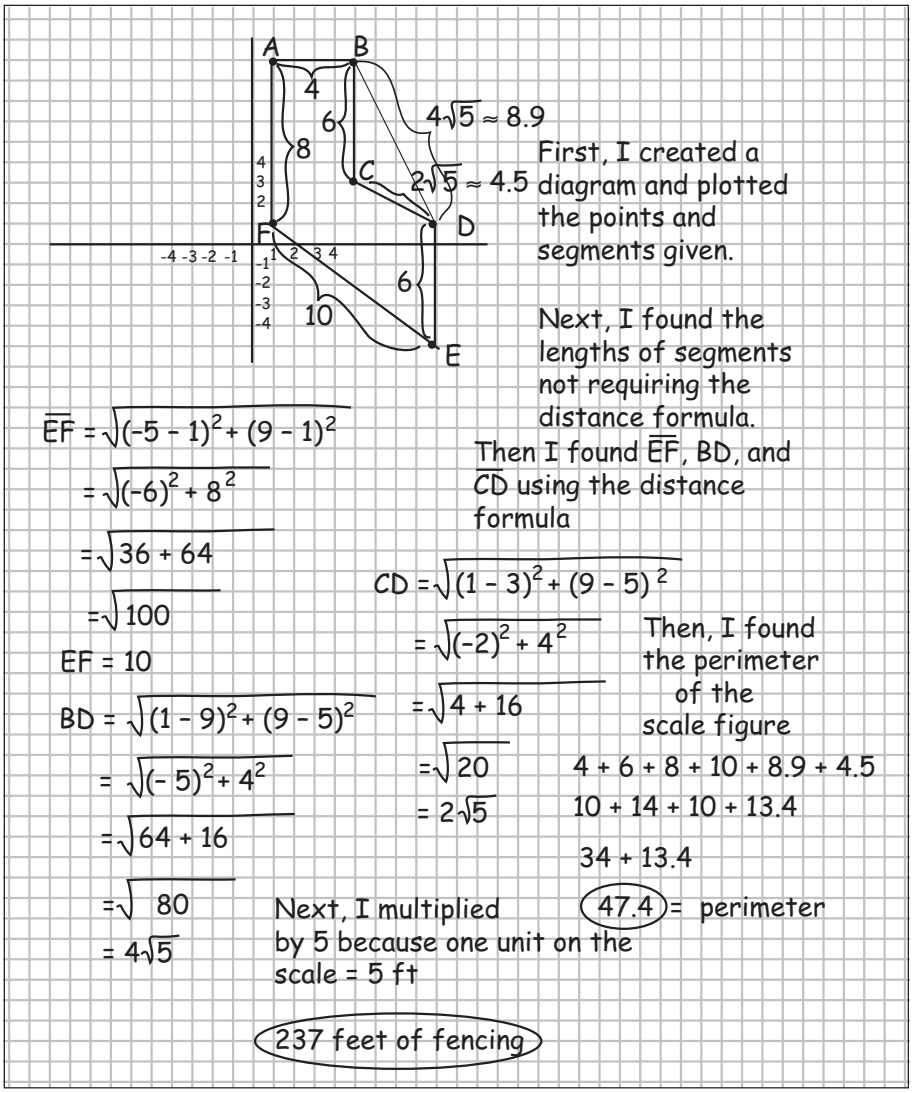
What Students Are Expected to Accomplish Mathematically

Student responses should include a correct answer of 237 feet for the perimeter. Responses must demonstrate the use of a correct scaling factor, use of the Pythagorean theorem or distance formula, and the addition of the side lengths to find the perimeter. Students must clearly communicate and accurately apply the process.

Sample Student Work for Geometry

Conceptual Understanding: 4
Mathematical Computation and Accuracy: 4
Communication: 3

STUDENT RESPONSE*



COMMENTARY

The student response shows a thorough understanding of the central mathematical ideas as demonstrated by using a correctly identified and applied process. The response includes the use of the distance formula to obtain the lengths of $\overline{EF}$, $\overline{BD}$, and $\overline{CD}$, multiplication by five to address the scaling factor, and addition of the side lengths to obtain the total perimeter. The work for the measures of each side is appropriately represented, with a minor error in referring to $\overline{EF}$.

The student response shows thorough use of mathematical skills by accurately demonstrating all required mathematical computations.

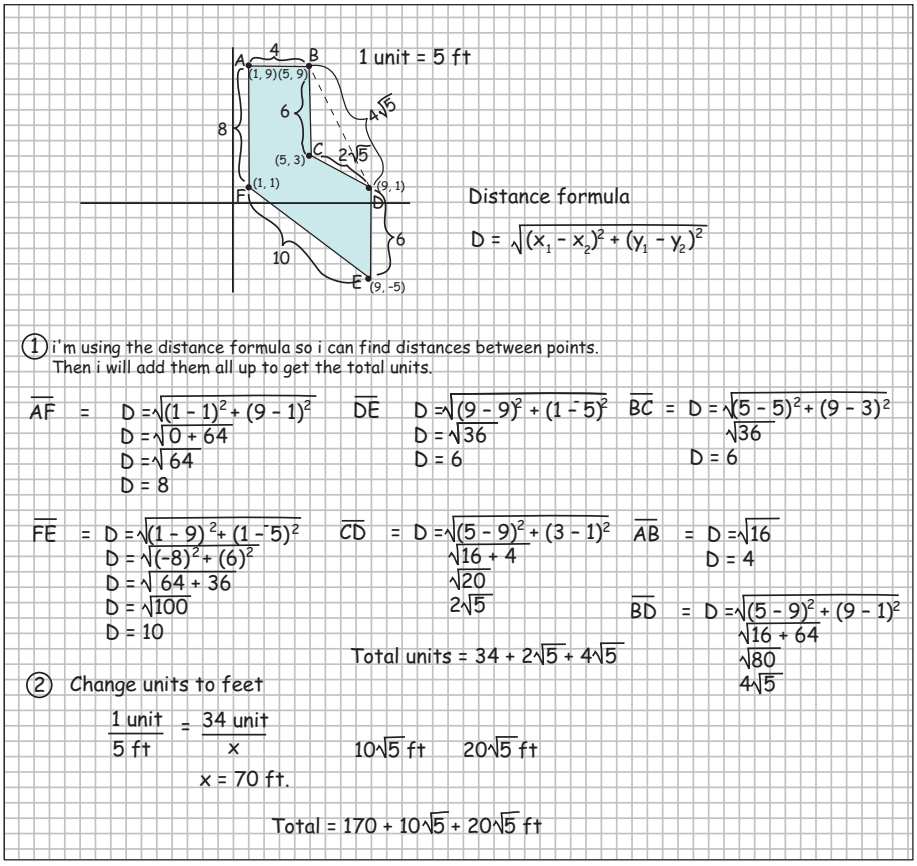
The student response shows clear organization with supporting evidence using appropriate mathematical language and reasoning. The response demonstrates clear communication of the three mathematical ideas required for this prompt.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Conceptual Understanding: 4
Mathematical Computation and Accuracy: 4
Communication: 3

STUDENT RESPONSE*



COMMENTARY

The student response shows a thorough understanding of the central mathematical ideas as demonstrated by using a correctly identified and applied process. The response includes the use of the distance formula to find the lengths of $\overline{AF}$, $\overline{DE}$, $\overline{BC}$, $\overline{FE}$, $\overline{CD}$, $\overline{AB}$, and $\overline{BD}$, and adding the appropriate measures to obtain the perimeter. The response uses proportions to address the scaling factor.

The student response shows thorough use of mathematical skills by accurately demonstrating all required mathematical computations. The total perimeter is correct, but not in simplified form.

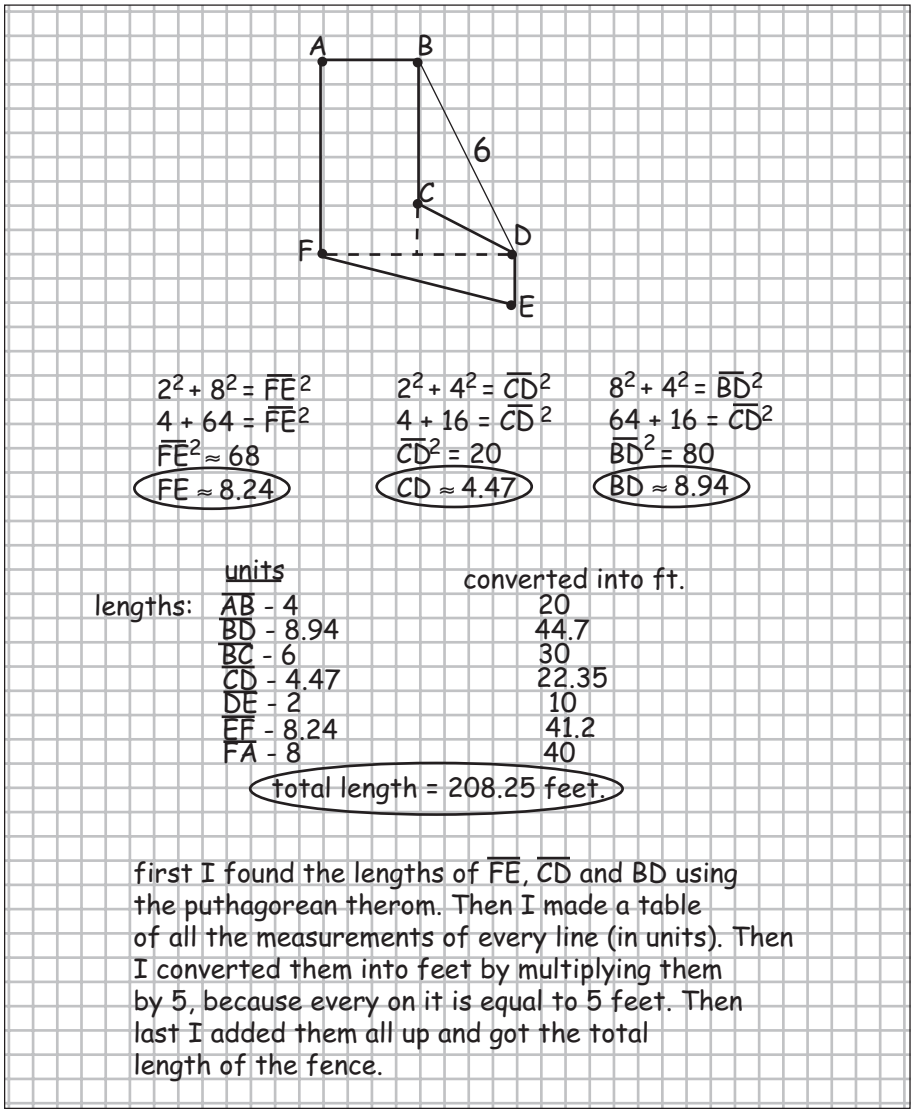
The student response shows clear organization with supporting evidence using appropriate mathematical language and reasoning.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Conceptual Understanding: 4
Mathematical Computation and Accuracy: 3
Communication: 3

STUDENT RESPONSE*



COMMENTARY

The student response shows a thorough understanding of the central mathematical ideas as demonstrated by using a correctly identified and applied process. The response includes the use of the Pythagorean Theorem to obtain the lengths $\overline{FE}$, $\overline{CD}$, and $\overline{BD}$, multiplication by five to address the scaling factor to correctly convert to feet, and addition of the side lengths to obtain a total perimeter.

The student response shows substantial use of mathematical skills. The response has point *E* incorrectly plotted, but uses the information to accurately demonstrate all required mathematical computations.

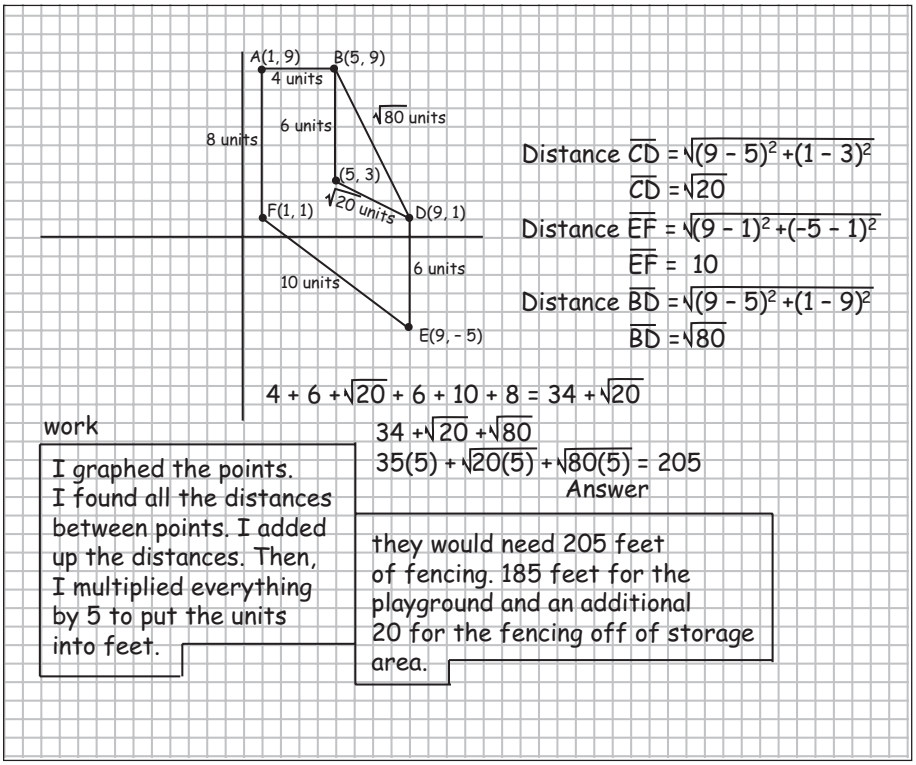
The student response shows clear organization with supporting evidence using appropriate mathematical language and reasoning. The communication addresses all three central mathematical ideas required for this prompt.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Conceptual Understanding: 4
Mathematical Computation and Accuracy: 2
Communication: 3

STUDENT RESPONSE*



COMMENTARY

The student response shows a thorough understanding of the central mathematical ideas as demonstrated by using a correctly identified and applied process. The response includes the use of the distance formula, multiplication by five to address the scaling factor, and addition of the side lengths to obtain the total perimeter.

The student response shows partial accuracy in required mathematical computations. The response accurately uses the distance formula to calculate the lengths of $\overline{CD}$, $\overline{EF}$, and $\overline{BD}$ and adds the side lengths to get the total perimeter. When multiplying by five to address the scale factor, the response performs an inaccurate calculation by using the five as a factor under the radical and by multiplying the radicals. This multiplication under the radical sign does not allow the required demonstration of the skill of adding radicals.

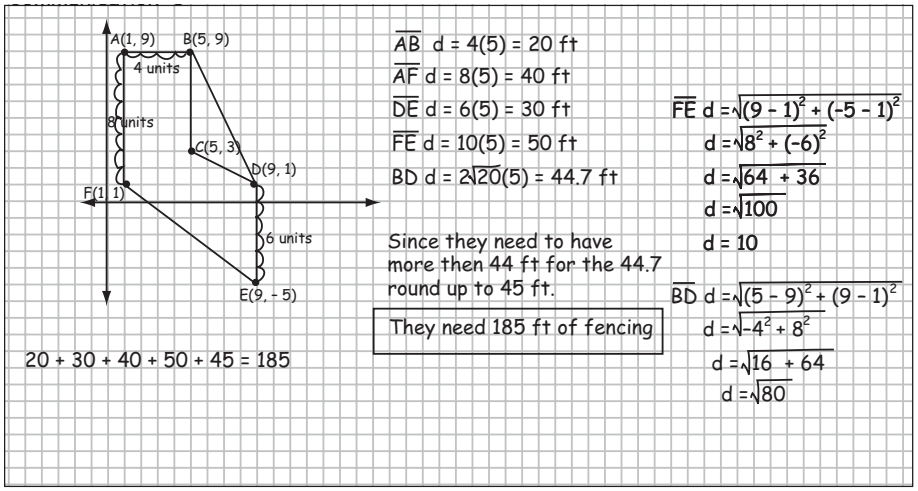
The student response shows clear organization with supporting evidence using appropriate mathematical language and reasoning.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Conceptual Understanding: 3
Mathematical Computation and Accuracy: 3
Communication: 3

STUDENT RESPONSE*



COMMENTARY

The student response shows substantial understanding of the central mathematical ideas as identified by demonstrating an appropriate process that contains a flawed application. The response includes the use of the distance formula to obtain the lengths of $\overline{FE}$ and $\overline{BD}$, multiplication by five to address the scaling factor, and addition of the side lengths to obtain a total perimeter. When calculating the total perimeter the student omits $\overline{BC}$ and $\overline{CD}$, and only finds the perimeter of pentagon *ABDEF*.

The student response shows substantial use of mathematical skills. The response contains a mathematical error when calculating the length of $\overline{BD}$, $(-4)^2$ is written as -4^2. The student work accurately calculates the perimeter for the pentagon.

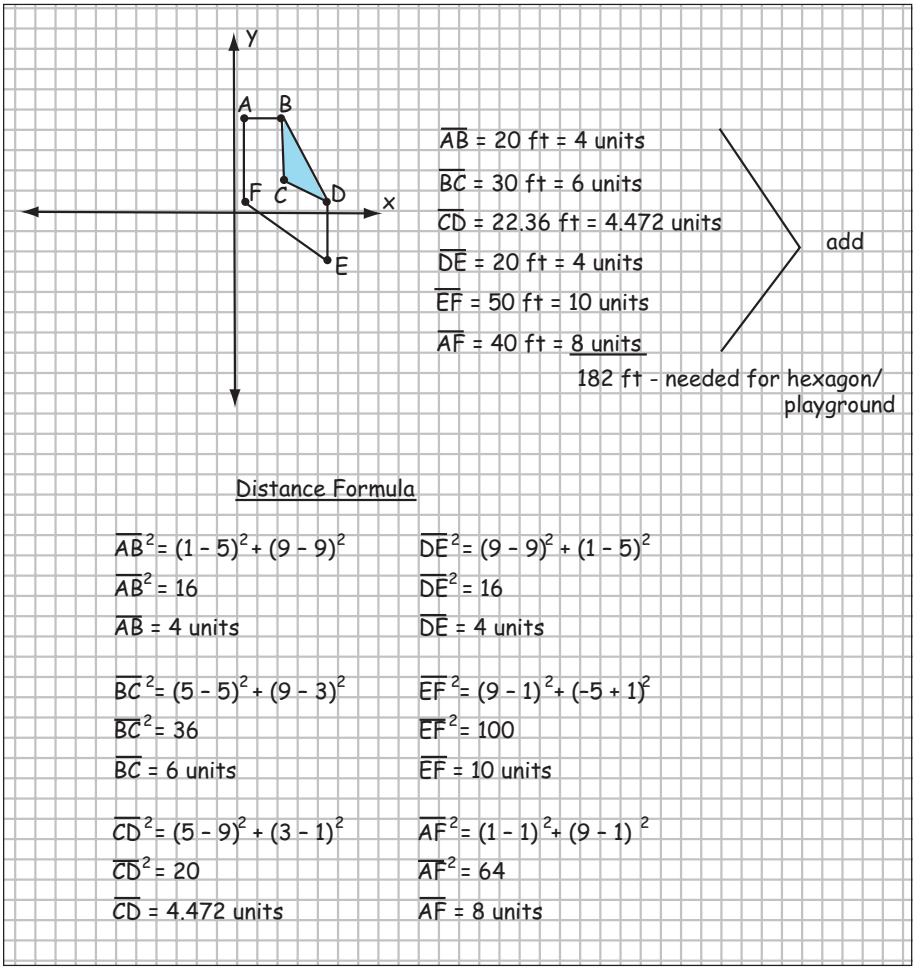
The student response shows clear organization with supporting evidence using appropriate mathematical language and reasoning.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Conceptual Understanding: 3
Mathematical Computation and Accuracy: 3
Communication: 3

STUDENT RESPONSE*



COMMENTARY

The student response shows substantial understanding of the central mathematical ideas by demonstrating an appropriate process that contains a flawed application. The response includes the use of the distance formula to obtain the lengths of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{AF}$, multiplication by five to address the scaling factor, and addition of the side lengths to obtain a total perimeter. When calculating the total perimeter, the student omits $\overline{BD}$ and only finds the perimeter of hexagon *ABCDEF*.

The student response shows substantial use of mathematical skills. The response contains a calculation error when determining the length of $\overline{DE}$. The student work uses the calculated values and finds the perimeter for the hexagon.

The student response shows clear organization with supporting evidence using appropriate mathematical language and reasoning. The student response demonstrates communication of the three mathematical ideas required for this prompt.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

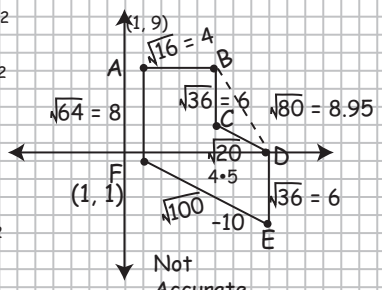
Conceptual Understanding: 3
Mathematical Computation and Accuracy: 2
Communication: 2

STUDENT RESPONSE*

$d^2 = (y_1 - y_2)^2 + (x_1 - x_2)^2$

$AB = (9 - 9)^2 + (1 - 5)^2$

$AB = 16$



$BC = (9 - 3)^2 + (5 - 5)^2$

$BC = 6^2$

$BC = 36$

$CD = (3 - 1)^2 + (5 - 9)^2$

$2 + 16$

$CD = 20$

$DE = (1 + 5)^2 + (9 - 9)^2$

$DE = 36$

$BD = (9 - 1)^2 + (5 - 9)^2$

$64 + 16$

$BD = 80$

$FE = (-5 + 1)^2 + (9 - 9)^2$

$36 + 64$

$FE = 100$

$AF^2 = (9 - 1)^2 + (1 - 1)^2$

8^2

$\sqrt{AF^2} = \sqrt{64}$

$AF = 8$

8
+ 4
12
+ 9
21
+ 6
27
+4.5
31.5
+24
55.5
x 5
277.5 ft of fencing needed

1 unit = 5 ft

I used the distance formula to find the perimeter of the hexagon. I added all the pieces together then multiplied by 5 because the scale was 1 unit = 5 feet on actual playground. I got 277.5 ft of fencing needed to circulate the palyground (55.5 ft x 5 = 277.5).

COMMENTARY

The student response shows substantial understanding of the central mathematical ideas by demonstrating an appropriate process that contains a flawed application. The response includes the use of the distance formula to obtain the lengths of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{BD}$, multiplication by five to address the scaling factor, and addition of the side lengths to obtain a total perimeter. When calculating the total perimeter the student adds several segments twice or has made an addition error to get the value of 24 as the final addend.

The student response shows partial accuracy in required mathematical computations. The response uses the distance formula to calculate the lengths of all the sides. The response does not take the square root of the value in the original calculation, but does so when labeling the length of the sides on the diagram. When calculating the total perimeter, the student adds several segments twice or has made an addition error to get the value of 24 as the final addend. The process for calculating the total perimeter contains a reference to equality where none exists.

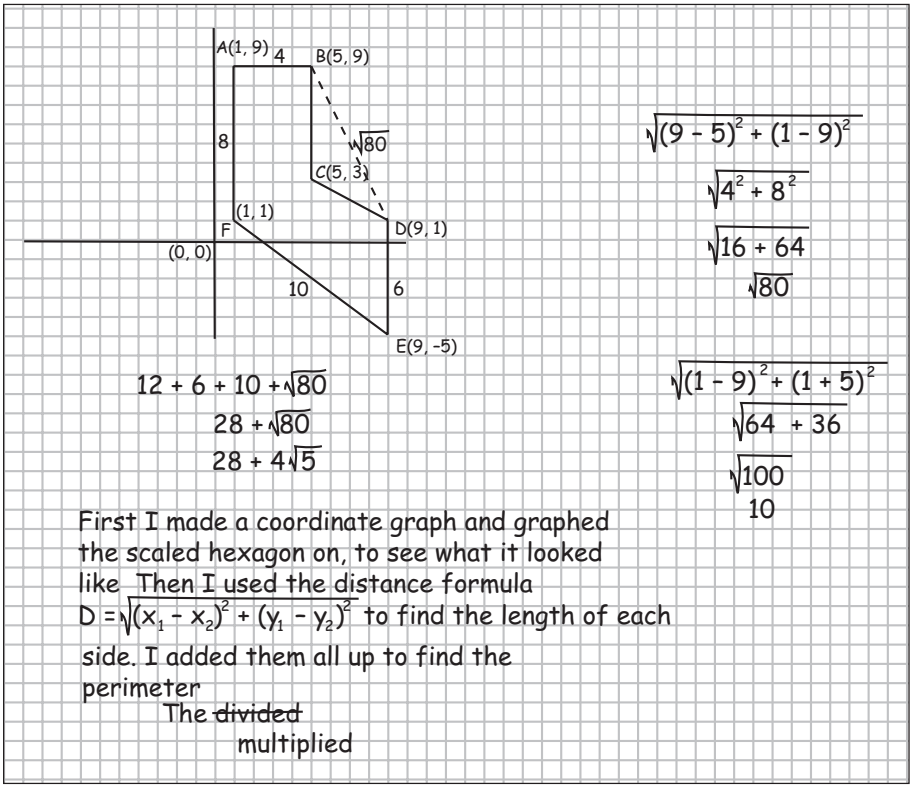
The student response shows some organization with supporting evidence. The student response demonstrates communication of the three mathematical ideas required for the prompt. The response also contains several statements that are unclear, such as where the 24 came from, why the square root was taken on the diagram (although it is not in the original calculation), and the calculated value of $\overline{DE}$ and $\overline{AF}$ do not agree with the diagram.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.

Sample Student Work for Geometry

Conceptual Understanding: 2
Mathematical Computation and Accuracy: 2
Communication: 2

STUDENT RESPONSE*



COMMENTARY

The student response shows a limited understanding of the central mathematical ideas by using the distance formula to calculate the length of $\overline{BD}$ and $\overline{EF}$ and by finding the perimeter of pentagon *ABDEF*. The scale factor and the additional piece of fencing are not addressed as required in the prompt.

The student response demonstrates partial accuracy in the required mathematical computations, but does not show calculations involving the scale factor. The response also does not indicate a final answer for the total perimeter.

The student response shows some organization with supporting evidence. The communication addresses two of the three central mathematical ideas, perimeter, and the use of the distance formula to find the lengths of two of the non-vertical or non-horizontal sides of the figure. There is no indication of what represents the total perimeter, and the rest of the communication is incomplete.

* The student response has been typed as written, with the student's own content, grammar, spelling, and punctuation.